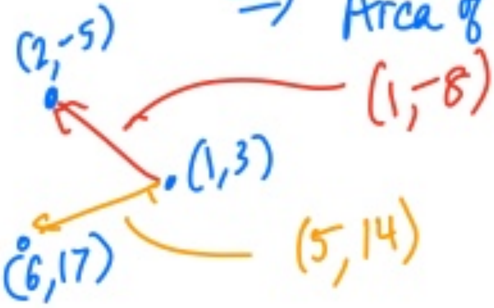
Example Plot $z = 2x^2 + 8y^2$

Examples from Review:

1.17: Area of triangle with vertices

$(2,-5), (1,3), (6,17)$.

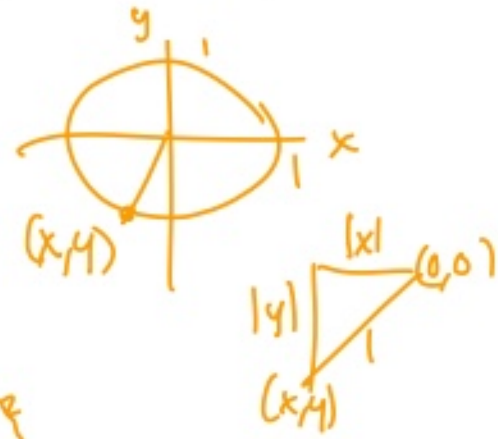
$\Rightarrow$ Area of triangle $= \frac{1}{2}$ Area of parallelogram



$$= \frac{1}{2}\left|\det\begin{pmatrix} 1 & -8 \\ 5 & 14 \end{pmatrix}\right| = \ldots$$

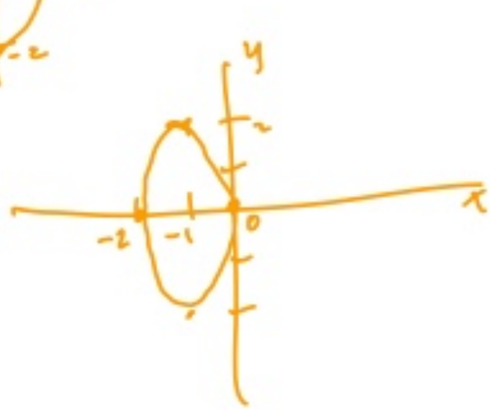
1.48 (d) $(x+1)^2 + \frac{(y+3)^2}{4} + (z-2)^2 = 1$

Practice: $x^2 + y^2 = 1$



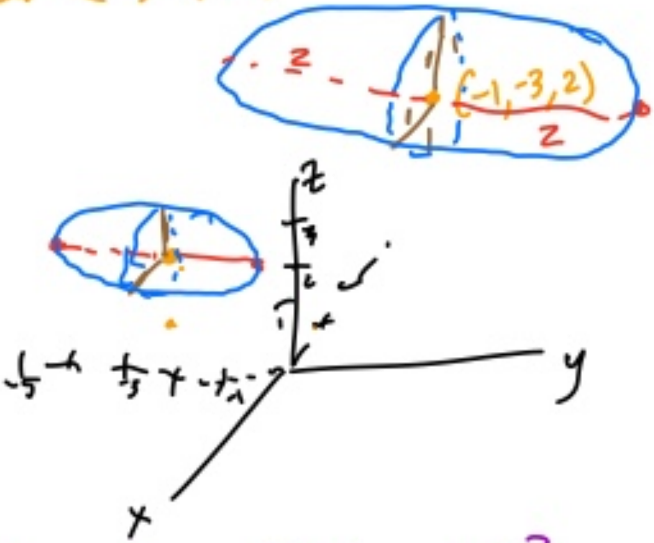
$x^2 + \frac{y^2}{4} = 1$

$(x+1)^2 + \frac{y^2}{4} = 1$



$$(x+1)^2 + \frac{(y+3)^2}{4} + (z-2)^2 = 1.$$

center $(-1,-3,2)$



$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} + \frac{(z-z_0)^2}{c^2} = 1$$

center (x_0, y_0, z_0)

major axes a, b, c

(1.50) Find eqn of quadric surface & graph.

cross section @ $z=2$ $\quad x^2 - 4x + 9y^2 = 14$

cross section @ $x=2$ $\quad y^2 - \frac{z^2}{4} = 1$

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quadric surface. $Ax^2 + Bx + Cy^2 + Dy + Ez^2 + Fz = G$

$z=2 \quad \underline{Ax^2 + Bx + Cy^2 + Dy} + 4E + 2F = G$

$x^2 - 4x + 9y^2 = 14 \longrightarrow G - 4E - 2F = 14$

$A=1, B=-4, C=9, D=0$

$x^2 - 4x + 9y^2 + Ez^2 + Fz = G$

$x=2 \quad \underbrace{4-8}_{-4} + 9y^2 + \underline{Ez^2 + Fz} = G$

$y^2 - \frac{z^2}{4} = 1 \quad \rightarrow$ constant $= G+4$

$\Rightarrow 9y^2 - \frac{9}{4}z^2 = 9$

$\Rightarrow E = \frac{-9}{4},\ F = 0 \quad 9 = G + 4$

$G = 5.$

$\Rightarrow$ Our equation is

$$x^2 - 4x + 9y^2 + \frac{-9}{4}z^2 = 5$$

Not in standard form.

$$x^2 - 4x + 4 + 9y^2 + \frac{-9}{4}z^2 = 5 + 4$$

$$\Rightarrow (x-2)^2 + 9y^2 - \frac{9}{4}z^2 = 9$$

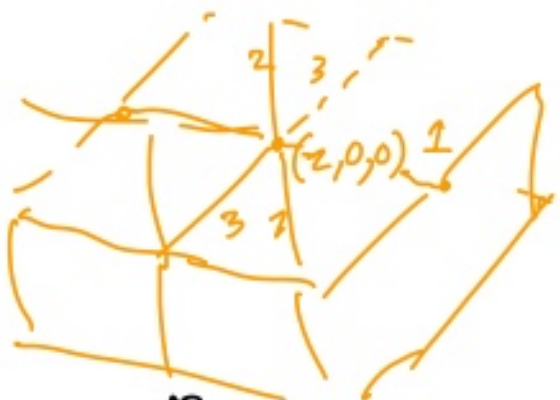
$$\Rightarrow \boxed{\frac{(x-2)^2}{9} + y^2 - \frac{z^2}{4} = 1}$$

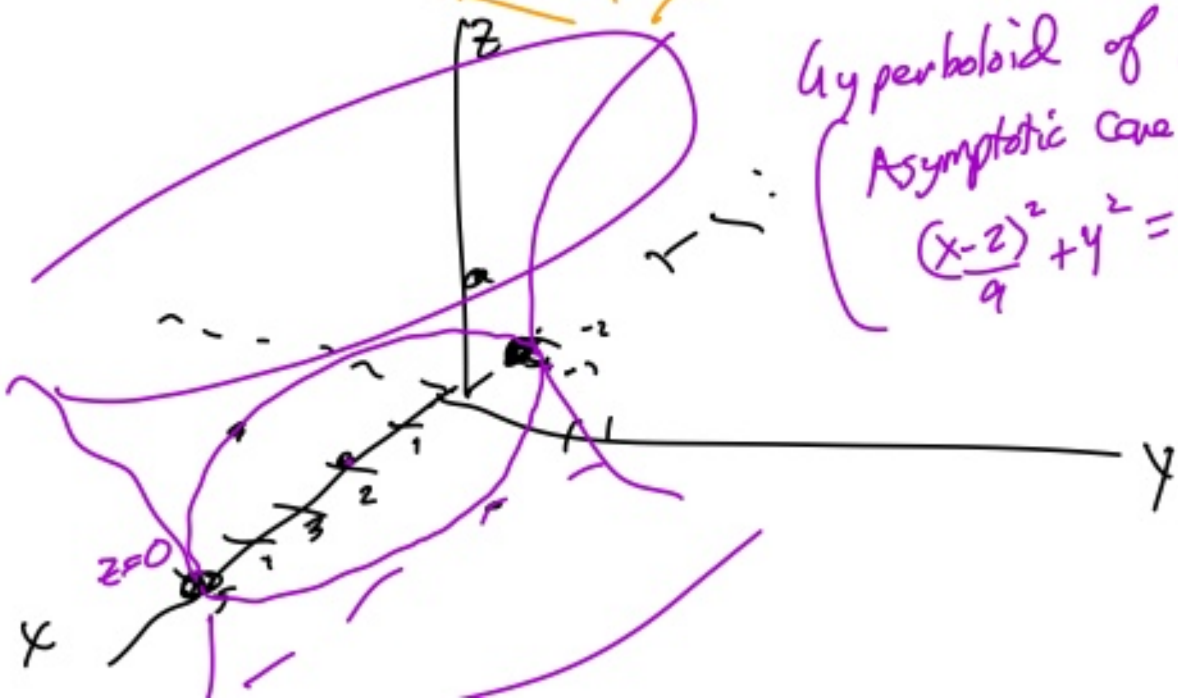
center $(2,0,0)$

axes $\underset{x}{3},\ \underset{y}{1},\ \underset{z}{2}$

$\frac{(x-2)^2}{9} + y^2 = 1 + \frac{z^2}{4}$

ellipse when z = constant.





Hyperboloid of 1 sheet.

Asymptotic Cone:

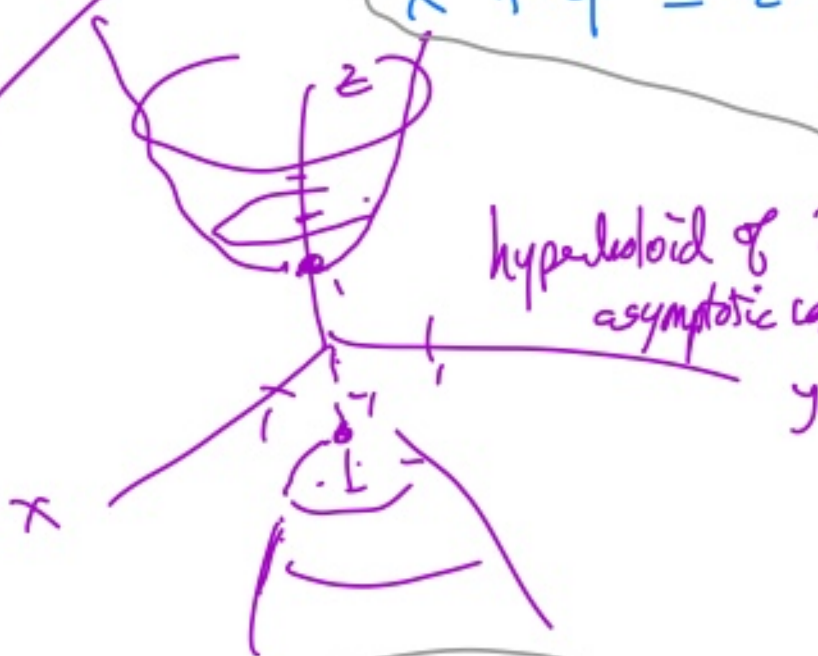
$$\frac{(x-2)^2}{9} + y^2 = \frac{z^2}{4}$$

$\boxed{\text{Eg}}$ graph $x^2+y^2=z^2-1$ ← $|z|\ge 1$

2 sheets

$x^2+y^2=z^2+1$ ← any z works

1 sheet.



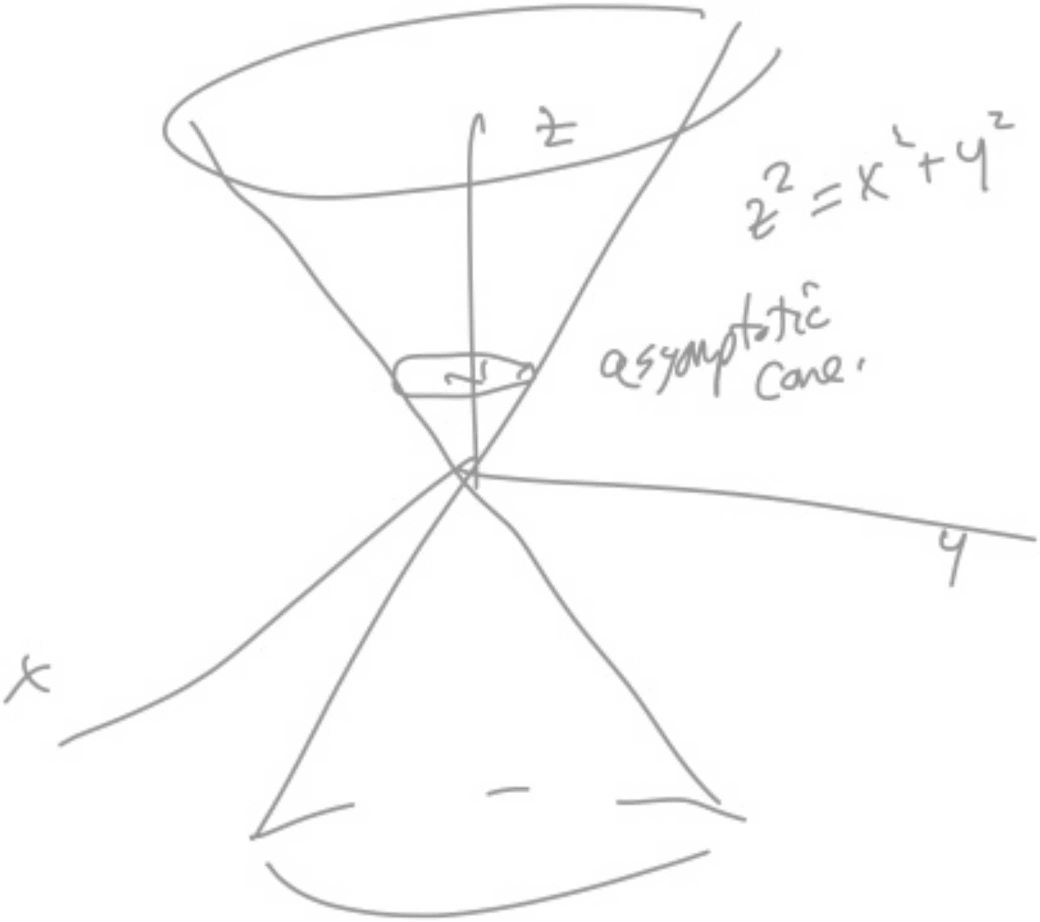
hyperboloid of 2 sheets

asymptotic cone $x^2+y^2=z^2$

hyperboloid of 1 sheet.

asymptotic cone

$x^2+y^2=z^2$



2.5 h

$$\beta(t) = (\overset{x}{t+3}, \overset{y}{t^2-1}, \overset{z}{2})$$

$t = x-3 \quad y = (x-3)^2 - 1$

Question: find points of greatest & least curvature.

velocity $\beta' = (1, 2t, 0)$

unit tangent vector $T = \dfrac{\beta'}{|\beta'|} = \dfrac{(1, 2t, 0)}{\sqrt{1+4t^2}} = \left(\dfrac{1}{\sqrt{1+4t^2}}, \dfrac{2t}{\sqrt{1+4t^2}}, 0\right)$

$= \left((1+4t^2)^{-1/2}, 2t(1+4t^2)^{-1/2}, 0\right)$

Curvature $= \dfrac{|T'|}{|\beta'|}$

$\to T' = \left(\frac{-1}{2}(1+4t^2)^{-3/2} \cdot 8t,\ 2(1+4t^2)^{-1/2} + 2t\left(\frac{1}{2}\right)(1+4t^2)^{-3/2} \cdot 8t,\ 0\right)$

$$\hookrightarrow T' = \left(\frac{-4t}{(1+4t^2)^{3/2}}, \frac{2}{\sqrt{1+4t^2}} - \frac{8t^2}{(1+4t^2)^{3/2}}, 0 \right)$$

$$T' = \left(\frac{-4t}{(1+4t^2)^{3/2}}, \frac{2+8t^2-8t^2}{(1+4t^2)^{3/2}}, 0 \right)$$

$$T' = (-4t, 2, 0) \cdot \frac{1}{(1+4t^2)^{3/2}}.$$

$$|\beta'| = \sqrt{1+4t^2}$$

$$\text{curvature} = K = \frac{|T'|}{|\beta'|} = \frac{\sqrt{16t^2+4}}{(1+4t^2)^2}$$

$(1+4t^2)^{1/2}(1+4t^2)^{3/2}$

$$= \frac{2\sqrt{1+4t^2}}{(1+4t^2)^2}$$

$$K = \frac{2}{(1+4t^2)^{3/2}}$$

minimum @ $\leftarrow$ No min. $K \to 0$ as $t \to \pm\infty$.

maximum @ $t=0$ $K=2$

point $(3,-1,2)$ max curvature

no min. curvature.

Like (1.32) Find the angle between the planes.

$$4x - 6y + 27z = 9$$

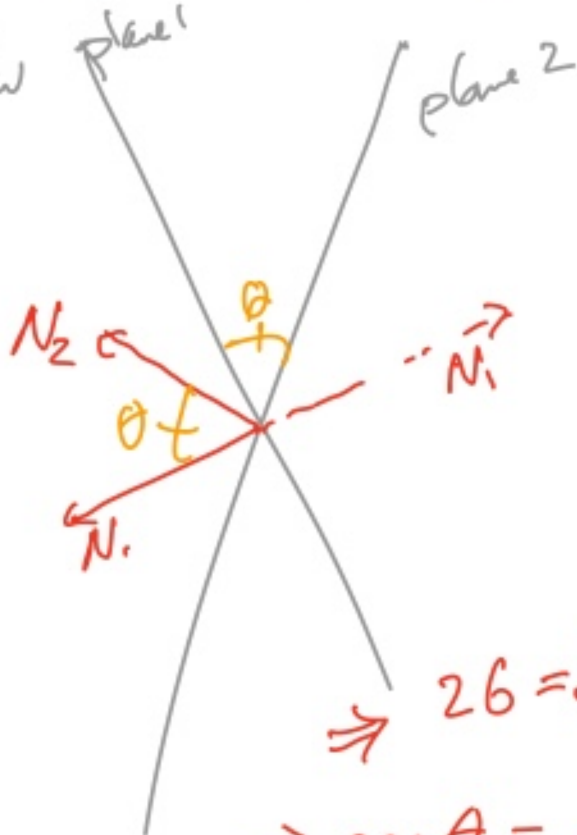
and

$$8x + 10y + 2z = 1.1111111\ldots$$

Normal vector: $(4, -6, 27)$

Normal vector: $(8, 10, 2)$

Side view



$$N_1 \cdot N_2 = |N_1|\,|N_2| \cos\theta$$

$$4 \cdot 8 + -6 \cdot 10 + 27 \cdot 2 = \underbrace{\sqrt{16 + 36 + 27^2}}_{\text{blah}} \underbrace{\sqrt{64 + 100 + 4}}_{\text{yaya}} \cos\theta$$

$$\Rightarrow 26 = \sqrt{\text{blah}}\,\sqrt{\text{yaya}}\,\cos\theta$$

$$\Rightarrow \cos\theta = \frac{26}{\sqrt{\text{blah}}\,\sqrt{\text{yaya}}}$$

$$\arccos\theta = \underline{\qquad\qquad}$$

Like 1.34 Find nonzero vector parallel

to $3.7x + 2y - 5.8z = 11.1$.

$\Longleftrightarrow$ Find a vector v s.t.

$v \cdot (3.7, 2, -5.8) = 0$

Let $v = (2, -3.7, 0)$ ← one example

or $(0, -5.8, -2)$

$(1, 1, \frac{+5.7}{5.8})$

or $(0, -5.8, -2) 285.$